Enroll No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER – 2016

M.Sc. Mathematics

16PMTCC01 - ALGEBRA -1

Duration of Exam – 3 hrs ***Semester – I*** ***Max. Marks – 70***

***Part A** (5x2= 10 marks)*

Answer **ALL** questions

1. What are all the subgroups of $(Z,+)$.

2. Decompose the following permutation into transpositions $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix}$

3. Is (Q,+) semi group?

4. Define: Characteristic of Ring and Integral Domain

5. In a field subtraction is closed yes or no? Give example

***Part B** (5X5 = 25 marks)*

Answer **ALL** questions

6a. A necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is $a \in H,\, b^{-1} \in H \Rightarrow ab^{-1} \in H$.

OR

6b. The intersection of any two normal subgroups of a group is a normal subgroup.

7a. Every group of prime order is a cyclic.

OR

7b. Let G be a group and $a \in G$. Then number of elements in the conjugate class $C(a)$ is equal to the index of the normaliser $N(a)$ of a in G .

8a. If $f: G \to G'$ is homomorphism with kernel H . For each subgroup K' of G' with $K = \{x \in G / f(x) \in K'\}$ then K is subgroup of G containing H and $K' \cong K/H$.

OR

8b. Show that every quotient group of a cyclic group is cyclic. Is converse true? Explain.

9a. Prove: A field has no proper ideals.

OR

9b. Prove: If a has multiplicative inverse in R then a is not a zero divisor.

10a. Prove: If p is pime then Z_p is a field.

OR

10b. Prove: D is an integral domain, then char D = 0 or char D is prime

<u>Part C</u> (5X7 = 35 marks)

Answer **<u>ALL</u>** questions

11a. State and prove fundamental theorem of homomorphism.

OR

11b.. If H be a normal subgroup of group G and K is a normal subgroup of group G containing H then $G/K \cong (G/H)/(K/H)$

12a. Suppose G is a group of finite order and p is a prime number. If $p^m \mid o(G)$ and p^{m+1} is not divisor of $o(G)$ then G has a subgroup of order p^m.

OR

12b. Let G be a group and let H be any subgroup of G. If N is any normal subgroup of group G then $HN/N \cong H/(H \cap N)$.

13a. Show that the set of all positive numbers forms an abelian group under the composition defined by $a*b = {(ab)}/{2}$.

OR

13b. State and prove Lagrange's theorem.

14a. Show that $Z[\sqrt{3}]$ is an integral domain

OR

14b. The set R={0,1,2,3,4,5} is a commutative ring with respect to '$+_6$' and '$\times_6$' as the two ring operations.

15a. C be the set of ordered pairs (a,b) of real numbers. Define addition and multiplication in C by the equations: (a,b) + (c,d) = (a+c, b+d) and (a,b).(c,d) = (ac-bd, bc+ad) then show that C is a field.

OR

15b. Show that for a field F, the set of all matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$ is a right ideal but not a left ideal of the ring R of all 2 x 2 matrices over the field F.